REFLECTION BY DEFECTIVE DIFFUSION BONDS

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ABSTRACT

The reflection of ultrasonic wave motion by planar defect distributions is investigated in this paper, with a view towards a method to detect and characterize bond-plane defects and with an ultimate objective of providing information on the bond strength of metal-to-metal bonds. The paper is primarily concerned with an approximate approach to analyze reflection by cavities and cracks which are distributed in a bond plane. The results show some of the characteristic features of reflection with variation of such parameters as angle of incidence, frequency, defect size, defect shape and defect spacing.

I. Introduction

Metal to metal bonds by the diffusion welding process are becoming more frequently used for the fabrication of complicated parts, such as integrally bladed compressor and turbine rotors in gas turbine engines. Diffusion bonds may, however, have three kinds of defects: volumetric defects (voids, inclusions), crack-like defects, and crystalline grains.

Defects, when they do occur, are generally distributed in the plane of the bond, and they tend to provide a plane of partial reflection for ultrasonic wave motion. Reflected and transmitted waves carry information on the nature of the defects, and hence the ultrasonic method can, in principle, be used to detect and characterize bond-plane defects. In this paper reflection and transmission by a planar distribution of one kind of defect, namely voids, is studied in some detail.

Reflection and transmission of time-harmonic waves by planar arrays of cracks, cavities and inclusions have been studied by Achenbach et al, [1]-[3], [7] and Thompson et al, [5]-[6]. Angel and Achenbach, [1], and Mikata and Achenbach, [2], have obtained numerically rigorous reflection and transmission coefficients for the two-dimensional problem of a periodic distribution of cracks. Nonperiodic crack distributions have been considered by Sotiropoulos and Achenbach [3]-[4]. Thompson and co-authors, [5]-[6], have studied reflection from imperfect bonds, both for one-dimensional and two-dimensional planar crack distributions based on the use of a distributed spring model. Reflection and transmission by a periodic array of spherical cavities has been investigated by Achenbach and Kitahara [7]. The relevant parameters for the present problem are the angle of incidence, the frequency, length measures of the spacing of the cavities and their sizes, and the properties of the material. Specific results are presented for reflection by a distribution of spherical cavities. For the special case of a periodic distribution the reflection and transmission coefficients computed as functions of the frequency agree very well with the exact numerical results of Ref. [7].

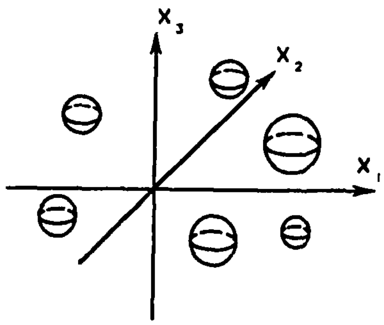

Fig. 1 Distribution of cavities in plane $x_3 = 0$

II. Problem Formulation

The plane defined by $x_3 = 0$ in an unbounded, homogeneous, isotropic, linearly elastic solid intersects a distribution of cavities of arbitrary shape. There are no cavities that are not intersected by the plane $x_3 = 0$. For the present discussion we consider an area of the plane $x_3 = 0$ defined by $|x_1| \le \ell$, $|x_2| \le w$. This area intersects M cavities, which are numbered m = 1, 2 ··· M. The geometry is shown in Fig. 1.

A plane longitudinal time-harmonic wave is incident on the distribution of cavities. The time-harmonic term $\exp(-i\omega t)$, where ω is the angular frequency, will however be omitted. The incident longitudinal wave is of the general form

$$\underline{u}^I = A\underline{d}^L \exp(ik_L \underline{p}^L \cdot \underline{x}) \ , \qquad (1)$$

Here A is the amplitude, and

$$\underline{d}^L = \underline{p}^L = (0, \sin\theta_L, \cos\theta_L) \quad . \qquad (2)$$

where θ_L is the angle of incidence. The surfaces of the cavities are free of tractions. Thus for the m-th cavity we have

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$$\underline{x} \in S_m: \quad t_i = \tau_{ij} n_j = 0 \quad , \qquad (3)$$

where τ_{ij} denotes the stress tensor, and $\underline{n}$ is the unit vector normal to the cavity surface S_m.

It is assumed that far from the plane intersecting the cavities ($x_3 = 0$), the dominant wave fields may be written as reflected and transmitted plane longitudinal and transverse waves of the forms:

$$\text{reflected:} \quad \underline{u}^{R\alpha} = R^\alpha A\, \underline{d}^{R\alpha} \exp(ik_\alpha \underline{p}^{R\alpha} \cdot \underline{x}) \qquad (4)$$

$$\text{transmitted:} \quad \underline{u}^{T\alpha} = T^\alpha A\, \underline{d}^{T\alpha} \exp(ik_\alpha \underline{p}^{T\alpha} \cdot \underline{x}) \qquad (5)$$

In R^α, we have α = L, TV and TH for reflected longitudinal, reflected vertically polarized transverse waves, and reflected horizontally polarized transverse waves, respectively. Analogous definitions hold for T^α, where α = L, TV and TH. The unit vectors appearing in Equation (4) are

$$\underline{p}^{RL} = (0, \sin\theta_L, -\cos\theta_L), \quad \underline{p}^{RTV} = \underline{p}^{RTH} = (0, \sin\theta_T, -\cos\theta_T) \qquad (6a,b)$$

$$\underline{d}^{RL} = \underline{p}^{RL}, \underline{d}^{RTH} = (1,0,0), \quad \underline{d}^{RTV} = (0, \cos\theta_T, \sin\theta_T) \qquad (7a,b,c)$$

The angles θ_L and θ_T are related by Snell's law:

$$k_L \sin\theta_L = k_T \sin\theta_T \qquad (8)$$

The unit vectors in Equation (5) are defined as

$$\underline{p}^{TL} = \underline{d}^{TL} = \underline{p}^L, \quad \underline{p}^{TTV} = \underline{p}^{TTH} = (0, \sin\theta_T, \cos\theta_T) \qquad (9a,b)$$

$$\underline{d}^{TTH} = (1,0,0), \quad \underline{d}^{TTV} = (0, -\cos\theta_T, \sin\theta_T) \qquad (10a,b)$$

Far from $x_3 = 0$ the total displacement fields may now be written as:

$$x_3 \geq 0: \ \underline{u}(\underline{x}) = \sum_\alpha T^\alpha A\, \underline{d}^{T\alpha} \exp(ik_\alpha \underline{p}^{T\alpha} \cdot \underline{x}), \quad \alpha = L, TV, TH \qquad (11)$$

$$x_3 \leq 0: \ \underline{u}(\underline{x}) = \underline{u}^I + \sum_\alpha R^\alpha A\, \underline{d}^{R\alpha} \exp(ik_\alpha \underline{p}^{R\alpha} \cdot \underline{x}), \quad \alpha = L, TV, TH \qquad (12)$$

Equations (11) and (12) imply that the plane containing the cavities is taken as a plane of homogeneous reflection and transmission. It has been shown by Achenbach and Kitahara [7] that this is rigorously correct for the lowest modes when the cavities are spheres and are periodically distributed in the plane $x_3 = 0$. For an arbitrary distribution of cavities Equations (11) and (12) are assumptions which should be good at low frequencies and at some distance from the plane $x_3 = 0$.

As shown by Achenbach and Kitahara [7] and subsequently in more general form by Sotiropoulos and Achenbach [3], the reflection coefficients R^α and the transmission coefficients T^α, can be obtained by application of the reciprocal identity. For details we refer to [3] and [7]. The results are

$$R^\alpha = \frac{1}{M} \sum_{m=1}^{M} \frac{1}{2} \frac{M}{\ell w} \frac{1}{\cos\theta_\alpha} \kappa_{ij}^{R\alpha} V_{ij}^{m,R\alpha} \qquad (13)$$

where α = L,TV,TH, ℓ and w are the length and the width of a rectangular area in the plane $x_3 = 0$, and M is the number of cavity intersections in that area. Also

$$T^L = 1 + \frac{1}{M} \sum_{m=1}^{M} \frac{1}{2} \frac{M}{\ell w} \frac{1}{\cos\theta_L} \kappa_{ij}^{TL} V_{ij}^{m,TL} \qquad (14)$$

$$T^{TV} = \frac{1}{M} \sum_{m=1}^{M} \frac{1}{2} \frac{M}{\ell w} \frac{1}{\cos\theta_T} \kappa_{ij}^{TTV} V_{ij}^{m,TTV} \qquad (15)$$

$$T^{TH} = \frac{1}{M} \sum_{m=1}^{M} \frac{1}{2} \frac{M}{\ell w} \frac{1}{\cos\theta_T} \kappa_{ij}^{TTH} V_{ij}^{m,TTH} \qquad (16)$$

Here $\kappa_{ij}^{R\alpha}$ and $\kappa_{ij}^{T\alpha}$ are constants for the m-th cavity and $V_{ij}^{m,R\alpha}$ and $V_{ij}^{m,T\alpha}$ are the cavity volume tensors of the m-th cavity. We have

$$V_{ij}^{m,R\alpha} = \int_{S_m} u_i n_j \exp(-ik_\alpha \underline{p}^{R\alpha} \cdot \underline{x}) dS \qquad (17)$$

with an analogous definition for $V_{ij}^{m,T\alpha}$, while

$$\kappa_{ij}^{RL} = -\frac{\lambda\delta_{ij} + 2\mu p_i^{RL} p_j^{RL}}{\lambda + 2\mu} \qquad (18)$$

and

$$\kappa_{ij}^{R\alpha} = p_i^{R\alpha} d_j^{R\alpha} + p_j^{R\alpha} d_i^{R\alpha} \quad \text{for} \quad \alpha = TV, TH, \qquad (19)$$

with analogous expressions for $\kappa_{ij}^{T\alpha}$.

III. Cavity Interaction Factor

Equations (13)-(16) show that the reflection and transmission coefficients for a planar distribution of cavities can be written in terms of the cavity volume tensors $V_{ij}^{m,\gamma\alpha}$ where γ = R (reflection) or γ = T (transmission) and α = L,TV or TH. An approximate calculation of the reflection and transmission coefficients can be based on the use of a cavity interaction factor to calculate the cavity volume tensors. Let us first consider the effect on reflection and transmission coefficients of the interaction between two neighboring cavities. These cavities are denoted by 1 and 2.

For cavity 1, the displacement components u_i^1 can be written as

$$u_i^1 = u_i^{10} + u_i^{12} \quad , \qquad (20)$$

where u_i^{10} are the displacement components induced by the scattered field in the absence of the second cavity, and u_i^{12} are the displacement components on cavity 1 due to the presence of cavity 2. The analogous components of the cavity volume tensor can be written as

$$V_{ij}^{1,\gamma\alpha} = V_{ij}^{10,\gamma\alpha} + V_{ij}^{12,\gamma\alpha} \qquad (21)$$

... V_{ij} ... is defined by Equation (17), and $V_{ij}^{10,\gamma\alpha}$ and $V_{ij}^{12,\gamma\alpha}$ are defined analogously. To determine $V^{12,\gamma\alpha}$ it is convenient to define the cavity interaction factor

$\beta_{ij}^{12,\gamma\alpha}$ by the relation

$$V_{ij}^{12,\gamma\alpha} = \beta_{ijmn}^{12,\gamma\alpha} V_{mn}^{2,\gamma\alpha} \tag{22}$$

The cavity interaction factor $\beta_{ijmn}^{12,\gamma\alpha}$ gives the components of the cavity volume tensor of cavity 1 due to the presence of cavity 2, in terms of the components of the total cavity volume tensor of cavity 2.

Details of the calculations of $V_{ij}^{10,\gamma\alpha}$ and $\beta_{ijmn}^{12,\gamma\alpha}$ are given by Xu and Achenbach [8]. The interaction factor $\beta_{ijmn}^{12,\gamma\alpha}$ will depend on a number of parameters. Prominent among these are of course the shape of cavity 1, the shape of cavity 2, the distance between the cavities as compared to their dimensions, the angle of incidence, and the frequency of the incident wave.

The concept of the interaction factor can now easily be extended to a distribution of cavities, i.e., for cavities k and s we define $\beta_{ijmn}^{ks,\gamma\alpha}$ by the relation

$$V_{ij}^{ks,\gamma\alpha} = \beta_{ijmn}^{ks,\gamma\alpha} V_{mn}^{s,\gamma\alpha} \tag{23}$$

where the summation is over m and n only. Then, with the additional assumption that $\beta_{ijmn}^{ks,\gamma\alpha}$ is not affected by the presence of cavities other than k and s, we can write

$$V_{ij}^{k,\gamma\alpha} = V_{ij}^{ko,\gamma\alpha} + \sum_{s=1(s\neq k)}^{M} \beta_{ijmn}^{ks,\gamma\alpha} V_{mn}^{s,\gamma\alpha} \tag{24}$$

Once the cavity interaction factors are kn[illegible] relatively simple way of calculating the components of the cavity volume tensor is by [illegible] use of an iteration procedure.

For cavity k the components of $V_{ij}^{k,\gamma\alpha}$ are given by Equation (24). In first approximation $V_{mn}^{s,\gamma\alpha}$ is taken as $V_{mn}^{so,\gamma\alpha}$, i.e., the value of $V_{mn}^{s,\gamma\alpha}$ in the absence of all the other cavities. The value of $V_{ij}^{k,\gamma\alpha}$ so obtained is termed $V_{ij}^{k1,\gamma\alpha}$, i.e.,

$$V_{ij}^{k1,\gamma\alpha} = V_{ij}^{ko,\gamma\alpha} + \sum_{s=1(s\neq k)}^{M} \beta_{ijmn}^{ks,\gamma\alpha} V_{mn}^{so,\gamma\alpha} \tag{25}$$

In the next step $V_{ij}^{s1,\gamma\alpha}$ is calculated by Equation (25), and the result is substituted in the right-hand side to yield

$$V_{ij}^{k2,\gamma\alpha} = V_{ij}^{ko,\gamma\alpha} + \sum_{s=1(s\neq k)} \beta_{ijmn}^{ks,\gamma\alpha} V_{mn}^{s1,\gamma\alpha} \tag{26}$$

This process can be repeated as many times as necessary. The procedure is particularly useful when the array of cavities is a periodic one, because then

$$V_{ij}^{s1,\gamma\alpha} = V_{ij}^{k1,\gamma\alpha} \tag{27}$$

is rigorously correct.

IV. Reflection by Spherical Cavities

The simplest result is achieved when the interaction between neighboring cavities is neglected. For that case the summation of terms on the right-hand side of Eq.(24) disappears. The expression for the coefficient of reflection, given by Eq.(13) reduces to

$$R^{\alpha} = \frac{\bar{M}}{2\cos\theta_{\alpha}} \kappa_{ij}^{R\alpha} \bar{V}_{ij}^{o,R\alpha} \tag{28}$$

Here $\bar{M} = M/\ell w$ denotes the number of cavity intersections per unit area in the plane $x_3 = 0$, and

$$\bar{V}_{ij}^{o,R\alpha} = \frac{1}{M} \sum_{m=1}^{M} V_{ij}^{o,R\alpha} \tag{29}$$

are the components of the averaged cavity colume tensor.

For a distribution of spherical cavities of radius d, and for normal incidence, $|R^L|$ according to Eq.(13) has been plotted versus $k_L d$ in Fig. 2. Here the Poisson's ratio of the material was selected as $\nu = 0.25$.

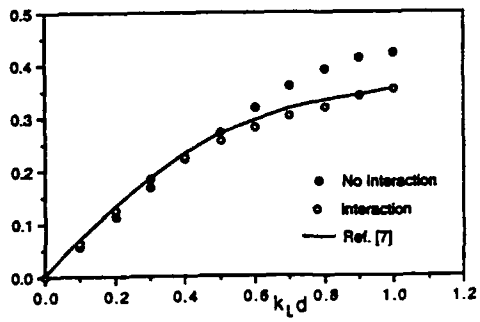

Fig. 2 Reflection coefficients for spherical cavities

It is of interest to compare $|R^L|$ for the case of no cavity interaction with corresponding results for the case where interaction has been taken into account. Achenbach and Kitahara [7] have presented rigorous results for reflection by a periodic array of spherical cavities whose centers are spaced distances a and b in the x_1 and x_2 directions, respectively. For $a/d = b/d = 3$, $|R^L|$ according to Ref.[7] has also been plotted in Fig. 2. It is noted that cavity interaction starts to

as the frequency becomes larger. Also plotted in Fig. 2 is the reflection coefficient for cavity interaction, but where the components of the cavity volume tensor have been computed according to the iteration procedure of Eqs.(25)-(27).

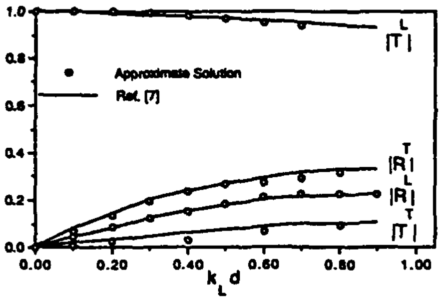

Fig. 3 Oblique incidence on periodic distribution of spherical cavities

Figure 3 compares the reflection and transmission coefficients for an angle of incidence $\theta_I = 30°$ computed by the method of this paper with the results of Ref.[7]. Good agreement is again observed.

V. Statistical Distribution of Cavities

Next we consider the effect on a central cavity of a random distribution of neighboring cavities. Let the neighboring cavities that have an effect on the central cavity be distributed in an annulus of Area A, defined by $a_{min} < r < a_{max}$, around the central cavity, see Fig. 4. The probability density function of the neighboring cavities is defined as

$$P(r,\psi) = \frac{1}{2\pi(a_{max} - a_{min})}, \tag{30}$$

i.e., the distribution is completely random in both the radial and circumferential directions.

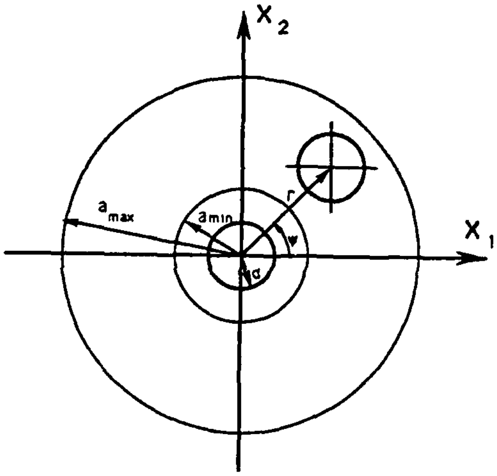

Fig. 4 Configuration of central cavity and neighboring cavity

in the usual manner. For example,

$$\langle\beta^{ks,\gamma\alpha}_{ijmn}\rangle = \frac{1}{2\pi(a_{max} - a_{min})}\int_0^{2\pi}\int_{a_{min}}^{a_{max}} \beta^{ks,\gamma\alpha}_{ijmn}(r,\psi)\,dr\,d\psi \tag{31}$$

The cavity interaction factor $\beta^{ks,\gamma\alpha}_{ijmn}$ naturally depends on the distance between the centers of the cavities. After an x_1x_2-system of axes to which the indices i,j,m and n refer, has been chosen, $\beta^{ks,\gamma\alpha}_{ijmn}$ will as also depend on the angle ψ between the line connecting the centers of two cavities and the x_1-axis, see Fig. 4. The relation between $\beta^{ks,\gamma\alpha}_{ijmn}(r,\psi)$ and $\beta^{ks,\gamma\alpha}_{ijmn}(r,\psi)$ is, however, of the following simple form:

$$\beta^{ks,\gamma\alpha}_{ijmn}(r,\psi) = Q_{si}(\psi)\beta^{ks,\gamma\alpha}_{stmn}(r,0)Q_{tj}(\psi) \tag{32}$$

where

$$[Q] = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{33}$$

Substitution of Eq.(32) into (31) allows a simple evaluation of the components of $<\beta^{ks,\gamma\alpha}_{ijmn}>$ in terms of $\beta^{ks,\gamma\alpha}_{stmn}(r,0)$.

In the next step of the argument it is realized that for a totally random distribution, every cavity can be considered as the central cavity, i.e., all cavities are the same. An application of the integral of Eq.(31) to Eq.(24) then leads to the relation

$$V^{k,\gamma\alpha}_{ij} = V^{ko,\gamma\alpha}_{ij} + (M-1) < \beta^{ks,\gamma\alpha}_{ijmn} > V^{s,\gamma\alpha}_{mn} \tag{34}$$

Here it has been taken into account that $V^{k,\gamma\alpha}_{mn}$ and $V^{ko,\gamma\alpha}_{ij}$ are independent of r and ψ. Also, M is the total number of cavities in the area πa^2_{max}.

Substitution of Eq.(34) into Eqs.(13) - (16) yields the corresponding expressions for the reflection and transmission coefficients. Numerical calculations have been carried out for specific choices of a_{min}, a_{max} and M. We also define the quantity ϵ as

$$\epsilon = \frac{Md^2}{a^2_{max}} \tag{35}$$

i.e., ϵ defines the ratio of the area of the cavity cross-sections to the total area that influences the central cavity.

For Poisson's ratio $\nu = 0.25$, $a_{min}/d = 2.8$ and $\epsilon = 0.126$, Fig. 5 shows the reflection and transmission coefficients, for the case of normal incidence.

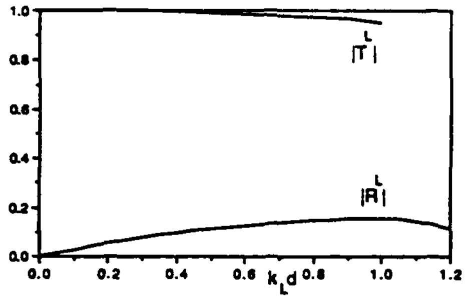

Fig. 5 Reflection and transmission coefficients for random distribution of spherical cavities

VII. Crack-Like Flaws

The analysis of this paper is also valid for crack-like flaws in the plane $x_3 = 0$. For crack-like flaws we have $n_1 = n_2 = 0$ and $n_3 = i_3$. The

expressions for the reflection and transmission coefficients become simpler in form, since

$$V_{11} = V_{22} = V_{31} = V_{32} = 0 \tag{36}$$

The remaining components of the cavity volume tensor now become components of the crack-opening tensor. Further details have been worked out in Ref.[4], where it has also been shown that the Mode-I stress-intensity factor for a distribution of equal-sized cracks can be directly related to reflection data.

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